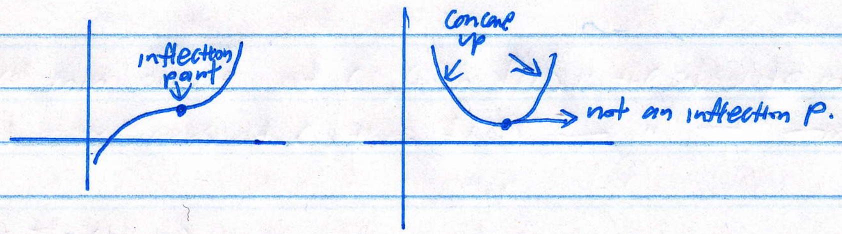
b) If f is continuous at $c \in I$ and f changes concavity, then f has an inflection point at c.



Eg : Determine the intervals on which the following functions are concave up or concave down. Identify the inflection points

$f(x) = 5x^4 - 20x^3 + 10$

$f'(x) = 20x^3 - 60x^3$

$f''(x) = 60x^2 - 120x = 60x(x-2)$

Theorem : (Test for concavity) Suppose f'' exists on an interval I where f is defined

- If $f'' > 0$ on I, then f is concave up.
- If $f'' < 0$ on I, f is concave down.
- If c is a point on I at which $f''(c) = 0$ and f'' changes sign at c, then f has an inflection point at c.

Test for concavity using second derivative

$f''(x) = 0 \Rightarrow x = 2$

$(-\infty, 0)$ $(0, 2)$ $(2, \infty)$

$f''(-1) > 0$ $f''(1) < 0$, $f''(3) > 0$

Conclude : f is concave up on $(\infty, 0)$ concave down on (concave down on $(0,2)$ and concave up on $(2, \infty)$